

DERIVATION OF TWO NUMERICAL METHODS TO CALCULATE DOUBLE INTEGRATIONS WITH THE CORRESPONDING CORRECTION LIMITS HAVING NUMERICAL CONTINUITY

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ABSTRACT

The main objective of this research is to derive two bases for calculating the values of double integers with continuous integrands and how to find the correction limits for them (error formula) and to improve these results using the Romberg acceleration method through the correction limits, when the number of partial periods is not equal in dimension s, x, y .

Keywords: Numerical integration 65D30 ; double integers 32A55 ; Romberg Accelerating 65B99 ; Taylor series 30K05.

1-INTRODUCTION

Numerical analysis is characterized by the creation of diverse methods for finding approximate solutions to certain mathematical issues in an effective manner. The efficiency of these methods depends on both the accuracy and the ease with which they can be implemented. Modern numerical analysis is the numerical interface of the broad field of applied analysis. The importance of bilateral integrals in finding the surface area and finding middle centers and the intrinsic limitations of flat surfaces and finding the volume under the surface of bilateral integration as the calculation of the area of the piece of ball $x^2 + y^2 + z^2 = 36$ Located inside the cylinder $y^2 + z^2 = 6y$ Frank Eyras[4].

In 2015, Fadhil [3] and Hassan[5] provided numerical methods for calculating the values of double integrals by applying the base point on the dimension x, y where the number of partial periods divided by the integration period on the inner, middle and outer dimension are not equal .

In 2016,Alsarefy [1]a numerical method was introduced to calculate single integrations and were of high accuracy.

In 2016, Hilal [6] introduced an numerical method to calculate a values of the doble integrals by using Rsim&M method which resulting from Rum brake's acceleration Sifi[7],with mid-point rule applied to the dimensions x ,y where dimensions are not equal.

In this research work on the derivation of the two methods by using mid- Point base and trapezoidal rule with Alsharify base [1]) to calculate the double integrals when the integration function continuously for periods of unequal, and we will divide the period of integration on the internal dimension x for a number of partial periods (n) and divide the period of integration on the external dimension y for a number of partial periods (m) , In a special case we will take(2 m=n), In other words (h₂=1/2h₁) Where h₁means the distances between the x coordinates and h₂is the distance between the coordinates y(

$h_1 = \frac{x_0 - x_n}{n}$ to improve the results on NT, Sifi[7], Rum brake's acceleration) We used $h_2 = \frac{y_0 - y_m}{2m}$, NN, taking advantage of the correction limits we obtained, and we symbolized the application of Rum brake's acceleration with the two rules with the symbol (RNN, RNT)

One of the rules we adopted in deriving the two methods is:

$$A = \int_{x_0}^{x_n} g(x)dx = \frac{h}{12} \left(5 \left(g_{x_0} + g_{x_n} \right) + 14 \sum_{i=1,3,5,\dots}^{n-1} g_i + 10 \sum_{i=2,4,6,\dots}^{n-2} g_i \right) + E(h) \dots (1)$$

Alsharify[1]

$$B = \int_a^b g(x)dx = h \sum_{j=1}^n \left[\frac{g(a) + g(b)}{2} \right] + E(h) \quad (2) \quad \text{Fox [2]}$$

$$C = \int_a^b g(x)dx = h \left(\frac{g_a + g_b}{2} + \sum_{i=1}^{n-1} g_i \right) + E(h) \quad \dots (3)$$

Fox [2]

When

$$E(h) = ah^2 + bh^4 + ch^6 + \dots$$

1.1 Evaluation of double integrals with Continuous Integrands numerically (NN)

Theorem

Let's $g(x, y)$ is a function is continuous and derivable at each point of the region $[x_0, x_n] \times [y_0, y_n]$ and the approximate value of the integration $I = \int_{y_0}^{y_n} \int_{x_0}^{x_n} g(x, y) dx dy$ can be calculated from the following rule

$$I = \int_{y_0}^{y_n} \int_{x_0}^{x_n} g(x, y) dx dy = \frac{h_1 h_2}{12} \sum_{i=0}^{n-1} [5(g(x_0, y_0 + ih_2 + h_2/2) + g(x_n, y_0 + ih_2 + h_2/2)) + 14 \sum_{j=1,3,\dots}^{n-1} g(x_0 + ih_1 + h_1/2, y_j) + 10 \sum_{j=2,4,\dots}^{n-2} g(x_0 + ih_1 + h_1/2, y_j) +$$

$$E(h_1, h_2) = (A_1 h_1^2 + B_1 h_1^4 + C_1 h_1^6 + \dots) + (A_1 h_2^2 + B_1 h_2^4 + C_1 h_2^6 + \dots)$$

Where A_i, B_i, C_i, \dots Constants depend on partial derivatives of the function $g(x, y)$

Just for each $i = 1, 2, \dots$.

Assume that integration I is defined as follows:

$$I = \int_{y_0}^{y_n} \int_{x_0}^{x_n} g(x, y) dx dy$$

Where $g(x, y)$ continuous integration at each point of the integration area $[x_0, x_n] \times [y_0, y_n]$

1.1. Evaluation of double integrals with Continuous Integrands numerically NN:

1.1.1 **A base consists of (base A on the internal dimension x and base B on the outer dimension y) where n (the number of partial periods divided by the period $[x_0, x_n]$) is equal to the weakness m (number of partial periods divided by the period $[y_0, y_n]$)**

it's mean $(n = 2m)$ and $\left(h = \frac{x_n - x_0}{n}\right)$.

We will mark the method with the symbol as RNN it R refers to the Rumbark acceleration Sifi[7] NN method and refers to the composite rule from the application of rule a on the internal dimension x and base b on the outer dimension y

In general, integration I can be written as follows:

$$I = \int_{y_0}^{y_n} \int_{x_0}^{x_n} g(x, y) dx dy = NN(h_1, h_2) + E(h_1, h_2) \quad \dots(1)$$

As representing $NN(h_1, h_2)$ the value of integration numerically using the base A on both dimensions, and that is the possible correction limits $E(h_1, h_2)$ added to the values $NN(h_1, h_2)$ of a string, and $\left(h_1 = \frac{x_n - x_0}{n} = \frac{y_n - y_0}{2m} \right)$.

For internal integration $\int_{x_0}^{x_n} g(x, y) dt$, it can be numerically calculated by rule A on the dimension X and (dealing with Y a constant).

$$\int_{y_0}^{y_n} \int_{x_0}^{x_n} g(x, y) dx dy = \int_{y_0}^{y_n} A dy = \int_{y_0}^{y_n} \frac{h_1}{12} \left[5(g(x_n, y) + g(x_0, y)) + 14 \sum_{i=1,3,\dots}^{n-1} g(x_i, y) + 10 \sum_{i=2,4,\dots}^{n-2} g(x_i, y) \right] dy + \sum_{i=1}^{\infty} \alpha_i (x_n - x_0) h_1^{2i} \frac{\partial^{2i} g(\mu_i, y)}{\partial x^{2i}} \quad (2)$$

To calculate the integration of the external dimension by applying rule B on each boundary of Equation 2 we obtain:

$$\text{a- } \int_{y_0}^{y_n} \frac{h_1}{12} (5g(x_n, y)) dy = \frac{h_1 h_2}{12} \sum_{i=0}^{m-1} 5(g(x_n, y_i + \frac{h_2}{2})) + h_2 \left(\int_{y_0}^{y_m} \left(\sum_{i=1}^{\infty} \alpha_i (x_n - x_0) h_1^{2i} \frac{\partial^{2i} g(\mu_i, y)}{\partial x^{2i}} \right) \right)$$

$$\text{b- } \int_{y_0}^{y_n} \frac{h_1}{12} (5g(x_0, y)) dy = \frac{h_1 h_2}{12} \sum_{i=0}^{m-1} 5(g(x_0, y_i + \frac{h_2}{2})) + h_2 \left(\int_{y_0}^{y_m} \left(\sum_{i=1}^{\infty} \alpha_i (x_n - x_0) h_1^{2i} \frac{\partial^{2i} g(\mu_i, y)}{\partial x^{2i}} \right) \right)$$

$$\text{c- } \int_{y_0}^{y_n} \frac{h_1}{12} \sum_{i=1,3,6}^{n-1} 14g(x_i, y) dy = \frac{h_1 h_2}{12} \sum_{j=0}^{m-1} \sum_{i=1,3,\dots}^{n-1} 14(g(x_i, y_j + \frac{h_2}{2})) + h_2 \left(\int_{y_0}^{y_m} \left(\sum_{i=1}^{\infty} \alpha_i (x_n - x_0) h_1^{2i} \frac{\partial^{2i} g(\mu_i, y)}{\partial x^{2i}} \right) \right)$$

$$d- \int_{y_0}^{y_m} \frac{h_1}{12} 10 \sum_{i=2,4,\dots}^{n-2} g(x_i, y) = \frac{h_1 h_2}{12} \sum_{j=0}^{m-1} \sum_{i=2,4,\dots}^{n-1} 10(g(x_i, y_j + \frac{h_2}{2})) + h_2 \left(\int_{y_0}^{y_m} \left(\sum_{i=1}^{\infty} \alpha_i (x_n - x_0) h_1^{2i} \frac{\partial^{2i} g(\mu_i, y)}{\partial x^{2i}} \right) \right)$$

Where as $\mu_i \dots \in (x_0, x_n) \ i = 1, 2, \dots$, $j = 0, 1, 2, \dots, n-1$, $y_j = y_0 + \frac{h_2}{2}$

$i=1, 2, 3, \dots, n-1$, $x_i = a + ih$

$i = 0, 1, 2, \dots, n-1$ $j = 1, 2, \dots, n-1$

By combining the equations of a ... b with the integration of the error formula in the equations we get the following

$$I = \int_{y_0}^{y_m} \int_{x_0}^{x_n} g(x, y) dx dy = \frac{h_1 h_2}{12} \sum_{i=0}^{n-1} [5(g(x_0, y_0 + ih_2 + h_2/2) + g(x_n, y_0 + ih_2 + h_2/2))$$

$$14 \sum_{j=1,3,\dots}^{n-1} g(x_j, y_0 + ih_2 + h_2/2) + 10 \sum_{j=2,4,\dots}^{n-2} g(x_j, y_0 + ih_2 + h_2/2)] +$$

$$h_2 \left(\int_{y_0}^{y_m} \left(\sum_{i=1}^{\infty} \alpha_i (x_n - x_0) h_1^{2i} \frac{\partial^{2i} g(\mu_i, y)}{\partial x^{2i}} \right) \right) + E(h_1, h_2)$$

Whereas

$$E(h_1, h_2) = h_2 \left(\int_{y_0}^{y_m} \left(\sum_{i=1}^{\infty} \alpha_i (x_n - x_0) h_1^{2i} \frac{\partial^{2i} g(\mu_i, y)}{\partial x^{2i}} \right) \right) =$$

$$E(h_1, h_2) = (x_n - x_0)(y_n - y_0) \left(h_2 \sum_{i=1}^{\infty} \alpha_i (x_n - x_0) h_1^{2i} \frac{\partial^{2i} g(\mu_i, \delta_i)}{\partial t^{2i}} \right) +$$

$$E(h_1, h_2) = (y_m - y_0)(x_n - x_0) \left[\alpha_1 h_1^2 \frac{\partial^2 g(\bar{n}_1, \bar{\mu}_1)}{\partial x^2} - \alpha_2 h_1^4 \frac{\partial^4 g(\bar{n}_2, \bar{\mu}_2)}{\partial x^4} + \dots \right] +$$

$$(y_m - y_0)(x_n - x_0) \left[\delta_1 h_2^2 \frac{\partial^2 g(\hat{n}_1, \hat{\mu}_1)}{\partial y^2} - \delta_2 h_2^4 \frac{\partial^4 f(\hat{n}_2, \hat{\mu}_2)}{\partial y^4} + \dots \right]$$

$$\begin{aligned}
 &= (y_m - y_0)(x_n - x_0) \delta_1 \left(\frac{h_1^2 \partial^2 g(\bar{n}_1, \bar{\mu}_1)}{\partial x^2} + \frac{h_2^2 \partial^2 g(\hat{n}_1, \hat{\mu}_1)}{\partial y^2} \right) + \\
 &\alpha_1 (y_m - y_0)(x_n - x_0) \left(\frac{h_1^4 \partial^4 g(\bar{n}_2, \bar{\mu}_2)}{\partial x^4} + \frac{h_2^4 \partial^4 g(\hat{n}_2, \hat{\mu}_2)}{\partial y^4} \right) + \dots \\
 &= (y_m - y_0)(x_n - x_0) \delta_1 \left(\frac{h_1^2 \partial^2 g(\bar{n}_1, \bar{\mu}_1)}{\partial x^2} + \frac{h_2^2 \partial^2 g(\hat{n}_1, \hat{\mu}_1)}{\partial y^2} \right) + \alpha_1 (y_m - y_0)(x_n - x_0) \left(\frac{h_1^4 \partial^4 g(\bar{n}_2, \bar{\mu}_2)}{\partial x^4} + \frac{h_2^4 \partial^4 g(\hat{n}_2, \hat{\mu}_2)}{\partial y^4} \right) + \dots
 \end{aligned}$$

Therefore, if the integral is a continuous function and its partial derivatives are present at each point of the integration zone $[x_0, x_n] \times [y_0, y_n]$, the error formula can be written as follows:

$$E(h_1, h_2) = (A_1 h_1^2 + B_1 h_1^4 + C_1 h_1^6 + \dots) + (A_1 h_2^2 + B_1 h_2^4 + C_1 h_2^6 + \dots)$$

Whereas

$$\begin{aligned}
 &(\bar{n}_\ell, \bar{\mu}_\ell), (\hat{n}_\ell, \hat{\mu}_\ell) \in [x_0, x_n] \times [y_0, y_m] \quad \text{and} \quad \delta_1, \delta_2, \dots \in (y_0, y_m), \alpha_1, \alpha_2, \dots \in (x_0, x_n) \quad i = 1, 2, 3, \dots \\
 &\alpha_i, \beta_i, A_1, A_2, \dots, B_1, B_2, \dots, C_1, C_2, \dots
 \end{aligned}$$

Constants depend on partial derivatives and do not depend on h.

a number of composite bases (base B on the internal dimension X and base a on the outer dimension Y)

In the same way we get proof

$$\begin{aligned}
 I &= \int_{y_0}^{y_n} \int_{x_0}^{x_n} g(x, y) dx dy = \frac{h_1 h_2}{12} \sum_{i=0}^{n-1} [5(g(x_0 + ih + h/2, y_0) + g(x_0 + ih + h/2, y_n)) \\
 &14 \sum_{j=1,3,\dots}^{n-1} g(x_0 + ih + h/2, y_j) + 10 \sum_{j=2,4,\dots}^{n-2} g(x_0 + ih + h/2, y_j)] + \\
 E(h_1, h_2) &= (A_1 h_1^2 + B_1 h_1^4 + C_1 h_1^6 + \dots) + (A_1 h_2^2 + B_1 h_2^4 + C_1 h_2^6 + \dots)
 \end{aligned}$$

Evaluation of double integrals with Continuous Integrands numerically(NT):

A composite base NT produced by the application (rule c on the dimension x and

Base A on the external dimensiony)

The method of proof is also similar to the method of NN but differs in the application of methods on the two dimensions where

$$I = \int_{y_0}^{y_n} \int_{x_0}^{x_n} g(x, y) dx dy = NT(h_1, h_2) + E(h_1, h_2) \tag{1}$$

For internal integration $\int_{x_0}^{x_n} g(x, y) dt$, it can be numerically calculated as base C on the dimension X and (dealing with Y a constant), And then calculate the integration on the second dimension y on each boundary of the integration using the rule A and dealing with the dimension X constant and using the theory of intermediate value in the differential we get:

$$I = \int_{y_0}^{y_n} \int_{x_0}^{x_n} g(x, y) dx dy = \frac{h_1 h_2}{24} [(5(g(x_0, y_0) + g(x_n, y_0) + g(x_n, y_n) + g(x_0, y_n))) + 14 \sum_{i=1,3,5}^{n-1} (g(x_0, y_i) + g(x_n, y_i)) + 10 \sum_{i=2,4,6}^{n-2} (g(x_0, y_i) + g(x_n, y_i)) + 2(\sum_{i=1}^{n-1} 5(g(x_i, y_0) + g(x_i, y_n))) + 14 \sum_{j=1,3,5}^{n-1} g(x_i, y_j) + 10 \sum_{j=2,4,6}^{n-2} g(x_i, y_j))] + (A_1 h_1^2 + B_1 h_1^4 + C_1 h_1^6 + \dots) + (A_1 h_2^2 + B_1 h_2^4 + C_1 h_2^6 + \dots)$$

Whereas $A_1, A_2, \dots, B_1, B_2, \dots, C_1, C_2, \dots$

Constants depend on partial derivatives and do not depend on the value of h.

2- EXAMPLE

$1 - I = \int_2^3 \int_2^3 \ln(\frac{x+y}{2}) dx dy$ and its analytical value 0.912930343651 rounded to 12 decimal places.

2- $I = \int_0^1 \int_0^1 \frac{x}{\sqrt{3-xy}} dx dy$ and its analytical value 0.3071345511905 rounded to 13 decimal places.

3- $I = \int_0^1 \int_0^1 \cos((xy)/2) dx dy$ And its analytical value 0.98621483608613 rounded to 14 decimal places.

4- $I = \int_0^1 \int_0^1 \cos((x+y)/2) dx dy$ And its analytical value 0.85945127165042 rounded to 14 decimal places.

Note:

The formal error for all example its:

$$E(h_1, h_2) = (A_1 h_1^2 + B_1 h_1^4 + C_1 h_1^6 + \dots) + (A_1 h_2^2 + B_1 h_2^4 + C_1 h_2^6 + \dots)$$

3-Results

1- $I = \int_2^3 \int_2^3 \ln\left(\frac{x+y}{2}\right) dx dy$ Here the integral is continuous integration in the region (2,3)

When comparing the analytical value of integration with its value in the Matlab program, we observe that it is perfectly identical "up to 14 after the interval $n = 2m = 64$ After using Rombrake Acceleration Sifi[7], with a correction limit of 2,4,6,8, ...) using the NN and NT methods while the value is correct for four decimal places using the NT rule without this acceleration

Accelerate and when using the base NN was correct value to five decimal places, note that "the time taken MATLAB account program was (1.3 minutes) As shown in Table 1 and 2. When compared to a way mid2 [5] Hassan, way Fadel [3] sim2 as well as the way Hilal [6] Rsim & M show that they have equal accuracy and speed of approaching the value of truth. With the time difference, where the roads we reached is the fastest.

Table1

n	m	NT	h=2	h=4	h=6	h=8
2	4	0.91197382573588				
4	8	0.91269116790727	0.91293028196439			
8	16	0.91287054678746	0.91293033974753	0.91293034359973		
16	32	0.91291539425202	0.91293034340688	0.91293034365084	0.91293034365165	
32	64	0.91292660629027	0.91293034363635	0.91293034365165	0.91293034365166	0.91293034365166

Table2

n	m	NN	h=2	h=4	h=6	h=8
2	4	0.91324868405297				
4	8	0.91301003758094	0.91293048875693			
8	16	0.91295027398012	0.91293035277985	0.91293034371471		
16	32	0.91293532666236	0.91293034422311	0.91293034365266	0.91293034365168	
32	64	0.91293158943114	0.91293034368739	0.91293034365168	0.91293034365166	0.91293034365166

$$2-I = \int_0^1 \int_0^1 \frac{x}{\sqrt{3-xy}} dx dy \text{ it's Continuously in the region for all } (x,y) \in [2,3] \times [2,3],$$

Through note (tables 3 and 4), we note that the results we have in the example 1 is similar using methods without accelerating, as well as after the use of accelerating as well as after comparing the results of the methods of researchers, the way mid2 [5] Hassan way, Fadel [3] sim2 as well as the way Hilal [6] Rsim & M, where they are equal in accuracy and the speed of approaching the value of truth. With the time difference, where the roads we reached is the fastest.

Tables 3

n	m	NT	h=2	h=4	h=6	h=8
2	4	0.98428422761809				
4	8	0.98573703674225	0.98622130645031			
8	16	0.98609568938019	0.98621524025950	0.98621483584678		
16	32	0.98618506835279	0.98621486134366	0.98621483608260	0.98621483608634	
32	64	0.98620739533671	0.98621483766468	0.98621483608608	0.98621483608613	0.98621483608613

Table4

n	m	NN	h=2	h=4	h=6	h=8
2	4	0.98686324922454				
4	8	0.98637440623531	0.98621145857223			
8	16	0.98625457061912	0.98621462541373	0.98621483653649		
16	32	0.98622475984892	0.98621482292552	0.98621483609298	0.98621483608594	
32	64	0.98621731641000	0.98621483526369	0.98621483608624	0.98621483608613	0.98621483608613

$$3-I = \int_0^1 \int_0^1 \frac{x}{\sqrt{3-xy}} dx dy \text{ it's Continuously in the region for all } (x,y) \in [0,1] \times [0,1] (x,y)$$

When comparing the analytical value of the integration with its value in the Matlab program (Table 5,6), we observe that it is perfectly identical "up to the 14th position after the interval after the use of Rumble acceleration Sifi[7], with the correction limits of 2.4,8,8 ...) using NN and NT methods The correct value for four decimal places using the NN, NT rules without this acceleration is to note that the time taken by the Matlab account was (60 seconds) as shown in Table (3). When compared to the mid2 [5] method and the method of fadel [3] sim2 as well as the method of crescent [6] Rsim & M are shown to be equal in accuracy and speed of approach to the real value. With the time difference where the methods we have reached are the fastest

Table5

n	m	NN	h=2	h=4	h=6	h=8
2	4	0.3058284294228				
4	8	0.3068082456864	0.3071348511076			
8	16	0.3070529906980	0.3071345723686	0.3071345537860		
16	32	0.3071141620880	0.3071345525514	0.3071345512302	0.3071345511897	
32	64	0.3071294539791	0.3071345512761	0.3071345511911	0.3071345511905	0.3071345511905

Table6

n	m	NT	h=2	h=4	h=6	h=8
2	4	0.3098931489268				
4	8	0.3078225980122	0.3071324143739			
8	16	0.3073064609651	0.3071344152828	0.3071345486767		
16	32	0.3071775222372	0.3071345426612	0.3071345511531	0.3071345511924	
32	64	0.3071452935520	0.3071345506569	0.3071345511899	0.3071345511905	0.3071345511905

4- $I = \int_0^1 \int_0^1 \text{Cos}(x+y)/2 dx dy$ it's Continuously in the region for all $(x,y) \in [0,1] \times [0,1]$ (x,y)

When the comparison between the analytical value of integration with its value in the MATLAB program (Table 7 and 8) note it's completely identical "until the fourteenth place after the interval when $n = 2m = 64$ after using the Rumbark's acceleration Sifi[7]up to correct 2,4,6 ...) using the NT method when using the method NN also but and when $n = 2m = 32$ the value was correct to four decimal places using the rules NN, NT without this acceleration of note "that the time taken for the account MATLAB program was (1 minute), as shown in table 5,6. When compared to a way mid2 [5] Hassan way Fadel [3] sim2 as well as the way Hilal [6] Rsim & M show that they have equal accuracy and speed of approaching the

Value of truth, with the time difference, where the roads we reached is the fastest.

Table 7

n	m	NN	h=2	h=4	h=6
2	4	0.86113295363895			
4	8	0.85987111693766	0.85945050470389		
8	16	0.85955619709029	0.85945122380783	0.85945127174810	
16	32	0.85947750076884	0.85945126866169	0.85945127165195	0.85945127165042

Table8

n	m	NT	h=2	h=4	h=6	h=8
2	4	0.85441410777487				
4	8	0.85819222778254	0.85945160111843			
8	16	0.85913652607051	0.85945129216650	0.85945127156970		
16	32	0.85937258621625	0.85945127293150	0.85945127164916	0.85945127165043	
32	64	0.85943160035192	0.85945127173047	0.85945127165040	0.85945127165042	0.85945127165042

CONCLUSION

We ate in this research calculate the approximate values of double integrals with continuous bases using a Al Sharifi Base [1] with the mid-point base,as well as the trapezoidal rule with the base to the Alsherif [1] on the internal and external dimensions x,yWhen the partial periods is not equal on both dimensions where we called NT, NN, and when you use them to Rumbark’s acceleration Sifi[7],our symbol (*RNN, RNT*).We conclude from the results and tables of this research that these methods give good results and high accuracy, that when using these rules NT, NN gave the correct results in $n = 2m = 32$,at $n = 2m = 64$ four decimal places relative to the analytical value and after the use of Rumbark’s acceleration Sifi[7], the value was rounded to fourteen decimal places in both methods, as in examples (4.3) and shortly.

In the two examples (1, 2), when using NN, we obtained a close value of five orders of magnitude. When using NT, the value obtained is four decimal places at $n = 2m = 64$. Thus we conclude that the two methods are of high accuracy and excellent speed in approaching the real value and can be relied upon in the calculation of double integrals.

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